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On Domain-like Structures in the QCD Vacuum

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We suggest that clusters or domains of topological charge and action density occur in the QCD vacuum as an effect of singularities in gauge fields and can simultaneously lead to confinement and chiral symmetry breaking. The string constant, condensates and topological susceptibility are estimated within a simplified model of hyperspherical domains with interiors of constant field strength with reasonable values obtained. Propagators of dynamical quarks and gluons have compact support in configuration space, thus having entire Fourier transforms, which gives rise to their confinement.

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Since Gribov and Singer [1], evidence has mounted that singularities in gauge vector potentials are intrinsically unavoidable in nonabelian theories. Several scenarios of confinement based on "condensation" of singular confirations such as monopoles and vortices, typical for the maximal abelian, Laplacian or axial gauges, are the subject of intensive analytical and numerical lattice studies today [2]. These focus mainly on the singular fields themselves, their emergence from gauge fixing, their topological properties and role in the QCD functional integral.

Here we emphasise a less studied effect due to singular gauge fields, namely their restrictive influence on fluctuations in the vicinity of singularities [3,4]. The subtleties of separating fields into regular and singular parts and the behaviour of regular fields at the singularities are irrelevant if one could calculate the QCD functional integral "exactly". But these issues become crucial if one undertakes approximations [4]. In gauge invariant quantities singularities due to ambiguities in gauge fixing should not occur. This can happen in the action either due to cancellations between derivative and commutor parts in the field strength if the singularity is topologically nontrivial (monopole or vortex) or due to finitness of both terms separately for topologically trivial singularities (domain walls, which, in general, can also appear [5]). The cancellation of singularities in the action density can be prohibited by fluctuations around them. Thus finiteness of the action implies specific constraints on fluctuations as is discussed in Ref. [4] where Polyakov gauge monopoles are considered.

We formulate a model partition function which incorporates singularities in gauge fields effectively via their restrictive effect on fluctuations. We assume that singularities are present in general in gauge potentials, and in their vicinity one can divide an arbitrary field A into singular pure gauge S and regular Q parts: $A_\mu = S_\mu + Q_\mu$. The field strength for pure gauge S vanishes, but the field strength and the action density for A is singular unless the field Q satisfies certain conditions in the vicinity of the singularities in S. To be explicit, at the cost of generality, we further assume that singularities in vector potentials are concentrated on hypersurfaces ∂V_j $(j = 1, \ldots, N)$ in Euclidean space of volume V, in the vicinity of which gauge fields can be divided as above into a sum of a singular pure gauge $S^{(j)}_\mu$ and regular fluctuation part $Q^{(j)}_\mu$, with a colour vector n^a_j associated with $S^{(j)}$. For such fields to have finite action the fluctuations charged with respect to n_j must obey specific conditions on ∂V_j. The interiors of these regions thus constitute "domains" V_j. Demanding finiteness of the classical action density, one arrives at

$$\breve{n}_j Q^{(j)}_\mu = 0,\ \psi = -i\,\not\eta^j e^{i\alpha_j\gamma_5}\psi,\ \bar\psi = \bar\psi i\,\not\eta^j e^{-i\alpha_j\gamma_5}, \quad (1)$$

for $x \in \partial V_j$, with the adjoint matrix $\breve{n}_j = T^a n^a_j$ in the condition for gluons, and a bag-like boundary condition for quarks, $\eta^j_\mu(x)$ being a unit vector normal to ∂V_j.

Equations (1) indicate that gauge modes neutral with respect to n^a_j are not restricted and provide for interactions between domains. In a given domain V_j the effect of fluctuations in the rest of the system is manifested by an external gauge field $B^a_{j\mu}$ neutral with respect to n^a_j. This enables an approximation in which domains are treated as decoupled but, simultaneously, a compensating mean field is introduced in their interiors. The model becomes analytically tractable if we consider spherical domains with fixed radius R and approximate the mean field in V_j by a covariantly constant (anti-)self-dual configuration with the field strength

$$\hat{\mathcal{B}}^{(j)}_{\mu\nu} = \hat n^{(j)} B^{(j)}_{\mu\nu},\ \tilde B^{(j)}_{\mu\nu} = \pm B^{(j)}_{\mu\nu},\ B^{(j)}_{\mu\nu} B^{(j)}_{\rho\nu} = B^2\delta_{\mu\rho},$$
$$\hat n^{(j)} = t^3\cos\xi_j + t^8\sin\xi_j\ ,\ \xi_j \in \{(2k+1)\pi/6\}^5_{k=0}, \quad (2)$$

where the parameter $B=\mathrm{const}$ is the same for all domains and the constant matrix $n^a_j t^a$ belongs to the Cartan subalgebra. Note that there is no source for this field on the boundary and therefore it should be treated as strictly homogeneous in all further calculations. The homogeneity itself appears as an approximation.

A self-consistent mean field approach requires calculation of the effective action as a functional of the mean field and characteristic functions of the domains. Its minima would give information about mean field character, shape and typical domain size. Needless to say that this problem has yet to be formulated. Nonetheless in Eqs. (2) we have already assumed the effective action favouring nonzero mean field strength parameter B and finite typical size R. With this and arbitrary constant mean field, it can be shown that the effective action for a domain exhibits twelve degenerate discrete minima corresponding to (anti-)self-dual configurations and six values (for $SU(3)$) of the angle ξ associated with the Weyl group. There is also a degeneracy in the orientation of the chromomagnetic field. The value $\xi_0=\pi/6$ is specific for an *ansatz* with the effective action polynomial in $\mathrm{Tr}\hat{\mathcal{B}}^k$, but the period $\pi/3$ is universal. Since the volume of the domain is finite the degenerate minima do not correspond to thermodynamical phases and have to be summed in the partition function. The partition function for the model is defined as

$$\mathcal{Z}=\mathcal{N}\lim_{V,N}\prod_{i=1}^{N}\int\limits_{V}\frac{d^4z_i}{V}\int\limits_{\Sigma}d\sigma_i\int\limits_{\mathcal{F}^i_Q}\mathcal{D}Q^i\int\limits_{\mathcal{F}^i_\psi}\mathcal{D}\psi_i\mathcal{D}\bar{\psi}_i\times$$
$$\delta[D(\breve{\mathcal{B}}^{(i)})Q^{(i)}]\Delta_{\mathrm{FP}}[\breve{\mathcal{B}}^{(i)},Q^{(i)}]e^{-S^{\mathrm{QCD}}_{V_i}\left[Q^{(i)}+\mathcal{B}^{(i)},\psi^{(i)},\bar{\psi}^{(i)}\right]},$$

where the thermodynamic limit assumes $V,N\to\infty$ with the density $v^{-1}=N/V$ taken finite. The fields $Q^{(i)}$, ψ_i and $\bar{\psi}_i$ are subject to boundary conditions Eq.(1), in which the original singularities are effectively encoded. Interaction between the original domains is substituted by the mean field. A background gauge condition is imposed. The integration measure $d\sigma_i$ is

$$\int\limits_{\Sigma}d\sigma_i\ldots=\frac{1}{48\pi^2}\int\limits_0^{2\pi}d\alpha_i\int\limits_0^{2\pi}d\varphi_i\int\limits_0^{\pi}d\theta_i\sin\theta_i\times \qquad (3)$$
$$\int\limits_0^{\pi}d\omega_i\sum_{k=0,1}\delta(\omega_i-\pi k)\int\limits_0^{2\pi}d\xi_i\sum_{l=0}^{5}\delta\left(\xi_i-(2l+1)\frac{\pi}{6}\right)\ldots.$$

Here φ_i and θ_i are spherical angles of the chromomagnetic field, ω_i is the angle between the chromomagnetic and chromoelectric fields, ξ_i is the angle in the colour matrix $\hat{n}_i$, α_i is the chiral angle and z_i is the centre of the domain V_i.

This partition function describes a statistical system of density v^{-1} composed of noninteracting clusters, each of which is characterised by a set of internal parameters and internal dynamics represented by the fluctuation fields. Correlation functions can be calculated taking the mean field into account explicitly and decomposing over the fluctuations. First of all we consider vacuum characteristics of the system in the zeroth order of this expansion.

The connected $n-$point correlator of the mean field strength tensor is found to be

$$\langle B^{a_1}_{\mu_1\nu_1}(x_1)\ldots B^{a_n}_{\mu_n\nu_n}(x_n)\rangle=B^n t^{a_1\ldots a_n}_{\mu_1,\ldots,\nu_n}\Xi_n(x_1,\ldots,x_n),$$
$$B^a_{\mu\nu}(x)=\sum_j^N n^{(j)a}B^{(j)}_{\mu\nu}\theta(1-(x-z_j)^2/R^2),$$
$$t^{a_1\ldots a_n}_{\mu_1,\ldots,\nu_n}=B^{-n}\int d\sigma_j n^{(j)a_1}\ldots n^{(j)a_n}B^{(j)}_{\mu_1\nu_1}\ldots B^{(j)}_{\mu_n\nu_n},$$
$$\Xi_n=\frac{1}{v}\int d^4z\theta\left(1-\frac{(x_1-z)^2}{R^2}\right)\ldots\theta\left(1-\frac{(x_n-z)^2}{R^2}\right),$$

by explicit calculation using the measure, Eq. (3). Translation-invariant functions $\Xi_n(x_1,\ldots,x_n)$ are equal to the volume of the overlap region for n hyperspheres of radius R and centres $(x_1,\ldots,x_n)$, normalised to the volume of a single hypersphere $v=\pi^2R^4/2$. They are continuous and vanish if $|x_i-x_j|\geq 2R$: correlations in the background field have finite range $2R$ and have no particle interpretation. The statistical ensemble of background fields is non-Gaussian since all connected correlators are independent and cannot be reduced to the two-point correlators. The simplest application of this is a gluon condensate

$$g^2\langle F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)\rangle=4B^2.$$

A significant vacuum parameter for the resolution of the $U_A(1)$ problem is the topological susceptibility [7]. First we consider the topological charge density for colour $SU(3)$ in this approximation

$$Q(x)=\frac{g^2}{32\pi^2}\tilde{F}F=\frac{B^2}{8\pi^2}\sum_{j=1}^{N}\theta[1-(x-z_j)^2/R^2]\cos\omega_j,$$

where $\omega_j\in\{0,\pi\}$ depends on the duality of the j-th domain. The topological charge is additive

$$Q=\int_V d^4xQ(x)=q(N_+-N_-),\ -Nq\leq Q\leq Nq,$$

where $q=B^2R^4/16$ is a 'unit' topological charge, namely the absolute value of the topological charge of a single domain, and N_+ (N_-) is the number of domains with (anti-)self-dual field, $N=N_++N_-$. The probability distribution for topological charge reads

$$\mathcal{P}_N(Q)=\frac{\mathcal{N}_N(Q)}{\mathcal{N}_N}=\frac{N!}{2^N\,(N/2-Q/2q)!\,(N/2+Q/2q)!},$$

where $\mathcal{N}_N(Q)$ is the number of configurations with a given charge and $\mathcal{N}_N$ is the total number of configurations. The distribution is symmetric about $Q=0$, where it has a maximum for N even. For N odd the maximum is at $Q=\pm q$. Averaged topological charge is zero. The topological susceptibility is then determined by the two-point correlator of topological charge density:

$$\chi=\int d^4x\langle Q(x)Q(0)\rangle=\frac{B^4R^4}{128\pi^2}.$$

Static quark confinement is examined via calculation of the Wilson loop

$$W(L)=\lim_{N\to\infty}\prod_{j=1}^{N}\int_V\frac{d^4z_j}{V}\int\frac{d\sigma_j}{N_c}\mathrm{Tr}e^{i\int_{S_L}d\sigma_{\mu\nu}(x)\hat B_{\mu\nu}(x)}.$$

Path ordering here is unnecessary as the matrices $\hat n^k$ belong to the Cartan subalgebra. For computational convenience we consider a circular contour in the (x_3,x_4) plane of radius L with centre at the origin. Calculation of the colour trace, integrating over spatial orientations of the mean field and positions of the domains and then taking the thermodynamic limit ($N\to\infty$, $v=V/N=\pi^2R^4/2$), gives an area law for a large Wilson loop $L\gg R$

$$W(L)=e^{\sigma\pi L^2+O(L)},\ \sigma=Bf(\pi BR^2),$$
$$f(z)=\frac{2}{3z}\left(3-\frac{\sqrt3}{2z}\int_0^{\frac{2z}{\sqrt3}}\frac{dx}{x}\sin x-\frac{2\sqrt3}{z}\int_0^{\frac{z}{\sqrt3}}\frac{dx}{x}\sin x\right).$$

The function f is positive for $z>0$ and has a maximum for $z=1.55\pi$. We choose this maximum to estimate the model parameters by fitting the string constant to the lattice result,

$$\sqrt B=947\mathrm{MeV},\ R^{-1}=760\mathrm{MeV},\tag{4}$$

with unit charge $q=0.15$, density $v^{-1}=42.3\mathrm{fm}^{-4}$ and the "observable" gluonic parameters of the vacuum

$$\sqrt\sigma=420\ \mathrm{MeV},\ \chi=(197\ \mathrm{MeV})^4,$$
$$(\alpha_s/\pi)\langle F^2\rangle=0.081\ \mathrm{GeV}^4.\tag{5}$$

This result indicates high density and strong background fields in the system. There is no separation of scales, $\sqrt BR\approx1$. Neither large domains nor stochasticity of background fields are seen here which *a posteriori* justifies the mean field averaging prescription in the partition function. The high density ensures area law dominance already at distances $2L\approx1.5-2$ fm. Taylor-expanding the integrand in the Wilson loop integral would reveal all n-point correlation functions of the background field, and arguments about the importance of a fast decay of correlators for static confinement [8] would then be seen to apply here.

Due to averaging over self- and anti-self-dual configurations and angles α_i there is no explicit violation of parity and chiral symmetry in the partition function. A spontaneous breaking can be tested by estimating the quark condensate. To first order in fluctuations this requires calculation of the quark propagator with the condition

$$i\,\not\eta(x)e^{i\alpha\gamma_5}S(x,y)=-S(x,y),\ (x-z)^2=R^2.$$

An analogous condition holds for $(y-z)^2=R^2$. Substitution (with the upper (lower) sign for (anti-)self-dual domains)

$$S=(i\not D+m)[P_\pm\mathcal H_0+P_\mp O_+\mathcal H_1+P_\mp O_-\mathcal H_{-1}],$$
$$O_\pm=[1\pm\hat n\vec\Sigma\vec B/|\hat n|B]/2,\ P_\pm=[1\pm\gamma_5]/2,$$

leads to equations for the scalar functions $\mathcal H_\zeta$:

$$(-D^2+m^2+2\zeta\hat B)\mathcal H_\zeta(x,y)=\delta(x,y),$$

where $\hat B=|\hat n|B$. Solutions vanishing at infinity would give a Green's function $\mathcal H_{-1}$ divergent in the massless limit due to zero modes of the Dirac operator. The bag-like boundary conditions remove zero eigenvalues from the spectrum, and the massless limit is regular.

To render our calculation transparent we choose $y=z=0$ and calculate the quark condensate at the domain centre. Then the general solutions for scalar Green's functions take the form ($\mu_\zeta=m^2/2\hat B+\zeta$)

$$\mathcal H_\zeta=\Delta(x^2|\mu_\zeta)+C_\zeta e^{-\hat Bx^2/4}M(1+\mu_\zeta,2,\hat Bx^2/2),$$

where $\Delta(x^2|\mu)$ is the vanishing at infinity scalar propagator for mass $2\hat B\mu$, and the second term is a homogeneous solution regular at $x^2=0$ expressed via the confluent hypergeometric function. The constants C_ζ can be fit to implement the boundary condition. The terms $m\mathcal H_0$ and $m\mathcal H_1$ vanish in the massless limit and do not contribute to the condensate. The nontrivial contribution comes from the term $m\mathcal H_{-1}$. The bag-like conditions imply that on the boundary $\mathcal H_{-1}$ satisfies a mixed condition with $f'=df/d|x|$ and the sign $(-)+$ corresponding to (anti-)self-dual domain,

$$2e^{\mp i\alpha}m\mathcal H_{-1}=-2\mathcal H'_{-1}-\hat BR^2\mathcal H_{-1},$$

which leads to the relations

$$\lim_{m\to0}m\mathcal H_{-1}=\frac{e^{\pm i\alpha}}{2\pi^2R^3}F(\hat BR^2/2)e^{-\hat Bx^2/4},$$
$$\mathrm{Tr}S(0,0)=\frac{e^{\pm i\alpha}}{2\pi^2R^3}\sum_{|\hat n|}F(\hat BR^2/2),$$
$$F(z)=e^z-z-1+\frac{z^2}{4}\int_0^\infty\frac{dte^{2t-z(\coth t-1)/2}}{\sinh^2t}(\coth t-1).$$

Note that the term in the propagator with nonzero trace has definite chirality correlated with the duality of domain and is proportional to the zero mode of the Dirac operator: $\not DP_\mp O_-\exp(-\hat Bx^2/4)=0$.

Averaging this result over α and (anti-)self-dual configurations and taking into account the α-dependence of the quark determinant [6]

$$\det S^{-1} \propto \exp\{i\alpha \int_v dx Q(x)\} \approx \exp\{\pm iq\alpha\},$$

with q being the unit topological charge, we get a finite result

$$\langle\bar{\psi}\psi\rangle = -\frac{q}{2\pi^2 R^3(1+q)} \sum_{|\hat{n}|} F(\hat{B}R^2/2) = -(228\mathrm{MeV})^3,$$

for B and R as determined above, Eq. (4). Obviously the condensate vanishes for zero topological charge q. Final conclusions require complete calculation of the condensate and detailed analysis of the eigenvalue problem.

Confinement of dynamical quarks and gluons can be studied in this context via the analytical properties of their propagators. As has been partly demonstrated for quarks, the propagators of quark and gluon fluctuations can be analytically calculated by reduction to the scalar problem, essentially that of a four-dimensional harmonic oscillator with total angular momentum coupled to the external field. The general solution is given by decomposition over hyperspherical harmonics. Qualitatively, the boundary conditions mean $x-$space propagators of charged fields are defined in regions of finite support where they have standard ultraviolet singularities. So their Fourier transforms are entire functions in the complex momentum plane [9]. This manifests confinement of dynamical fields [10,11]. A known consequence of entire propagators is a Regge spectrum of relativistic bound states [11].

The model displays the most essential features of QCD and, being analytically tractable, provides a framework for phenomenological applications. The first test in this direction should be a calculation of the spectrum of bound states via the Bethe-Salpeter framework, or within a hadronisation scheme analogous to [11]. The model preserves the structure of a relativistic quantum field theory, albeit a nonlocal one [12]. Certainly one has to be convinced that the qualitative arguments underlying the model are formally legitimate. Associating a colour direction with the boundary is a restrictive assumption. Somewhat akin to "abelian projection", it singles out a class of abelian mean fields, and "neutral" gluons are not confined. Incorporating topologically nontrivial singularities can improve this aspect of the model.

Clustering of action and topological charge density with dominance by (anti-)self-dual fields in the QCD vacuum is evident in lattice cooling algorithms, which, by design, iterate towards (anti-)instantons [13]. Alternately, Horváth et al. [14] suggested studying duality of clusters via the chirality of fermionic eigenmodes. Used with overlap fermions this shows strong correlation between clusters of topological charge and locations where several low eigenvalue fermionic modes are largest; the modes are also chiral [15]. This is regarded as evidence for instantons. Such correlation between chirality of fermionc modes and duality of domains is also evident in the model discussed above. Our estimates suggest that formation of clusters, predominantly (anti-)self-dual and with average size $2R \approx 0.5$fm, can have purely quantum origin whose explanation could require reference to the existence of obstructions in gauge fixing rather than to the quasi-classical limit.

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